Core Mathematics C4 For Edexcel
Advanced Level

Paper H
Time: 1 hour 30 minutes

Instructions and Information

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration.

Full marks may be obtained for answers to ALL questions.

The booklet 'Mathematical Formulae and Statistical Tables', available from Edexcel, may be used.

When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Advice to Candidates

You must show sufficient working to make your methods clear to an examiner.
Answers without working may gain no credit.

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1. (*a*) Using the trapezium rule, with two trapeziums, show that an estimate for

$$\int_{-1}^{1} \frac{1}{1+e^{-x}}\,dx \text{ is } 1.$$

(4)

(*b*) Use the substitution $u = e^x$ to show that the *exact* value of the same integral is 1. **(4)**

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2. (*a*) The equation of a curve is

$$x = e^y.$$

(i) Find an expression for $\frac{dy}{dx}$ in terms of x. **(2)**

(ii) Find the equation of the tangent to the curve at the point where $y = 0$. **(2)**

(*b*) For the curve $x = \sin y$, show that $\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$. **(3)**

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3. A curve has parametric equations

$$x = 2\sin\theta + 1, \quad y = 2\cos\theta + 2.$$

(*a*) Show that the equation of the tangent at the point with parameter θ is

$$x\sin\theta + y\cos\theta = 2 + 2\cos\theta + \sin\theta$$

(4)

(*b*) Write down the equation of the tangent at the point where $\theta = \frac{\pi}{2}$. **(1)**

(*c*) Find the cartesian equation of the curve. **(4)**

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4. Points on a curve C satisfy the differential equation

$$\frac{dy}{dx} = -\frac{x-2}{y+1}.$$

The point (2, 2) lies on C.

(*a*) Show that the equation of C may be written as

$$(x-2)^2 + (y+1)^2 = 9.$$

(6)

(*b*) Sketch the curve C. **(2)**

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5. A warm object is immersed in a cold liquid. At time t minutes its temperature θ°C is given by

$$\theta = 70\,\mathrm{e}^{-0.1t} + 2.$$

(a) Write down the initial value of θ. **(1)**

(b) Find the value of θ when $t = 10$. **(2)**

(c) State the value which the temperature of the object approaches after a long time. **(2)**

(d) Find the time taken for the temperature of the object to reach 10°C. **(3)**

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6. (a) Use the identity $\sin^2\theta + \cos^2\theta \equiv 1$ to prove that

$$1 + \tan^2\theta \equiv \sec^2\theta.$$

(2)

(b) Use the substitution $x = \tan\theta$ to show that

$$\int_{\frac{1}{\sqrt{3}}}^{1} \frac{1}{(1+x^2)}\,\mathrm{d}x = \frac{\pi}{12}.$$

(6)

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7. (a) Express

$$\frac{9x}{(1-2x)(1+x)^2}$$

in partial fractions. **(4)**

(b) Hence, or otherwise, find the first three terms in the expansion of $\dfrac{5x}{(1-2x)(1+x)^2}$ as a series in ascending powers of x. **(5)**

8.

Figure 1

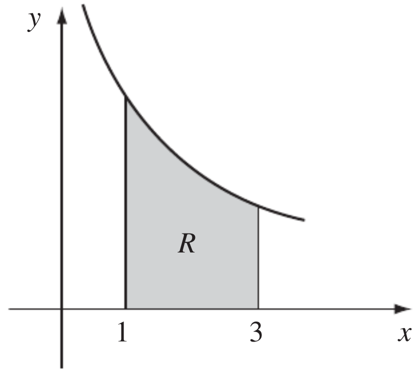

Figure 1 shows a sketch of the curve C with equation $y = \dfrac{2x+1}{x}$, $x \neq 0$.

The shaded region R is bounded by C, the x-axis and the lines $x = 1$ and $x = 3$.

(a) Find the area of the region R.

(3)

The region R is rotated through 360° about the x-axis to form a solid shape S.

(b) Show that the volume of S is $\pi\left(\dfrac{26}{3} + 4\ln 3\right)$.

(6)

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9. Points A and B have position vectors $\begin{pmatrix} 7 \\ 8 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 9 \\ 7 \\ 3 \end{pmatrix}$ respectively, relative to an origin O.

(a) Find a vector equation of the line through A and B in terms of a parameter λ.

(3)

(b) Calculate the acute angle between OA and AB, correct to the nearest degree.

(2)

(c) The point M on AB is such that OM is perpendicular to AB. Find the position vector of M.

(4)

END **TOTAL 75 MARKS**